ANANDALAYA

PRE BOARD EXAMINATION - 2

Class : XII

विद्या सवर्धि साधिका

Subject : MATHEMATICS M.M : 100

Date : 16 /01/2016 Time : 3 Hours

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 26 questions divided into 3 sections A, B and C. Section-A comprises of 6 questions of 1 mark each, Section-B comprises of 13 questions of 4 marks each and Section-C comprises of 7 questions of 6 marks each.

(iii)All questions in Section-A are to be answered in one word, one sentence or as per the exact requirements of the question.

(iv) There is no overall choice; however internal choice has been given in four questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables if required.

SECTION A

1. If $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$, then write the value of $(x + y)$. 1

2. Write the principle value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. 1

3. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{\imath}$, $\frac{\pi}{4}$ with $\hat{\jmath}$ and acute angle θ with $\hat{k}$, then find the value of θ. 1

4. Evaluate: $\int_{-\pi}^{\pi}(sin^{-93} x + x^{295})\, dx$. 1

5. * is a binary operation defined on Q, given by a*b = a + ab; a, b ∈Q. Is * commutative? 1

6. Solve: $\frac{dy}{dx} = e^{x-y} + x^3e^{-y}$. 1

SECTION B

7. Show that $A = \begin{bmatrix}2 & -3\\3 & 4\end{bmatrix}$ satisfies the equation $A^2 - 6A + 17I = O$, hence find the A^{-1}. 4

8. Draw and sketch of the following region and find its area: 4

$$\{(x, y)|\ \ x^2 + y^2 \le 1 \le x + y\}.$$

OR

Sketch the region enclosed between the circles $x^2 + y^2 = 1\ amd\ x^2 + (y-1)^2 = 1$.Also find the area of the region using integration.

9. Find : $\int_0^2 |x^2 + 2x - 3|\, dx$. 4

10. Prove that: $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$. 4

OR

Solve for *x*, $2\tan^{-1}(sinx) = \tan^{-1}(2sec\, x),\ x \ne \frac{\pi}{2}$.

12. Find the points on the curve $\boldsymbol{y = x^3 - 2x^2 - 2x}$ at which the tangent lines are parallel to the line $\boldsymbol{y = 2x - 3}$. 4

13. A random variable X has the following probability distribution: 4

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find the value of (i) k (ii) P(X < 3) (iii) P(X > 6) (iv) P(0 < X 3).

14. Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$a + d = b + c on the set N x N is an equivalence relation. 4

15. A company has two plant to manufacture T.V.s. The first plant manufactures 70% of the T.V.s and the rest are manufactured by the other plant. 80% of the T.V.s manufactured by the first plant are rated of standard quality, while that of the second plant only 70% are of standard quality. If a T.V. chosen are random is found t be of standard quality, find the probability that it was produced by the first plant. 4

16. Express the following matrix as the sum of a symmetric and a skew symmetric matrix and verify your result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ 4

OR

Obtain the inverse of the given matrix, using elementary operations: $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

17. A farmer decides to plant up to 10 hectares with cabbages and potatoes. He decides to grow at least 2 but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. If he can make a profit of Rs. 1500 per hectare on cabbages and Rs. 2000 per hectare on potatoes, how should he plan his farming so as to get the maximum profit? 4

18. Using properties of determinants prove the following, 4

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

19. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0. 4

OR

Find the coordinates of the point, where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersects the plane x – y + z – 5 = 0. Also find the angle between the line and the plane.

SECTION C

20. Find the largest possible area of a right angled triangle whose hypotenuse is 5cm long. 6

21. Form the differential equation of the family of circles in the second quadrant and touching the coordinate`s axes 6

OR

Show that the differential equation $2y\, e^{\frac{x}{y}}\, dx + \left(y - 2x\, e^{\frac{x}{y}}\right) dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$.

22. A sign board in the shape of intersection of parabolas $\boldsymbol{y = 6x - x^2}$ and $\boldsymbol{y = x^2 - 2x}$ On the sign board life values such as **"OBEDIENT"**, **"OSERVANT"**, **"EMPATHY"**, **"SINCERE"** etc., are to be written. What is the area of the sign board? What do you think about acquiring value **"EMPATHY"**? 6

23. If $\overrightarrow{\alpha} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\overrightarrow{\beta} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$, then express $\overrightarrow{\beta}$ in the form $\overrightarrow{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$, where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$. 6

24. Differentiate with respect to x: $y = \tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$ 6

OR

(i) Verify Rolle's Theorem for $f(x) = \sin x + \cos x \quad in \left[0, \frac{\pi}{2}\right]$.

(ii) Verify the Lagrange's mean value theorem for $(x) = x^2 + 3x + 3 \; in \; [4, 6]$.

25. Find the equation of the planes through the intersection of the planes $\boldsymbol{x + 3y + 6 = 0}$ and $\boldsymbol{3x - y - 4z = 0}$ whose perpendicular distance from the origin is equal to 1. 6

26. Evaluate: $\int cosec^3 x \;\; dx.$ 6