ANANDALAYA
FIRST PRE-BOARD EXAMINATION
CLASS –XII

Subject : MATHEMATICS M.M : 100
Date : 04 /12 /2015 Time : 3 Hours

General Instructions:

(i) All questions are compulsory.
(ii) The question paper consists of 26 questions divided into 3 sections A, B and C. Section-A comprises of 6 questions of 1 mark each, Section-B comprises of 13 questions of 4 marks each and Section-C comprises of 7 questions of 6 marks each.
(iii) All questions in Section-A are to be answered in one word, one sentence or as per the exact requirements of the question.
(iv) There is no overall choice; however internal choice has been given in four questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted. You may ask for logarithmic tables if required.

SECTION –A

1 If A is a square matrix and $|A| = 2$, then write the value of $|A\,A^T|$, where A^T is the transpose of the matrix A.

2 Write the order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}$.

3 Find the area of a parallelogram whose diagonals are $\overrightarrow{d_1} = 5\hat{\imath}$ and $\overrightarrow{d_2} = 2\hat{\jmath}$.

4 Evaluate: $\int \frac{sec^2\sqrt{x}}{\sqrt{x}}\,dx$.

5 A line in xy- plane makes angle $\frac{\pi}{6}$ with the x- axis. Find the direction cosines of the line.

6 If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$, then find $|\vec{a} - \vec{b}|$.

SECTION –B

7 Evaluate: $\int \left[\log(logx) + \frac{1}{(logx)^2}\right] dx$.

8 Using properties of integrals, evaluate: $\int_0^{\pi/2}(2log\ sinx - logsin2x)\,dx$

OR

Using properties of integrals, evaluate: $\int_0^{\pi/2} \frac{sin^2x}{sinx + cosx}\,dx$

9 Find the value of λ so that the four points with position vectors $-6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$, $3\hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$, $5\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$, $-13\hat{\imath} + 17\hat{\jmath} - \hat{k}$ are coplanar.

10 Find the equation of the line passing through the point (1, 3, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$.

11 A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all three try to solve the problem simultaneously, find the probability that exactly one of them can solve it.

12 A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. 6000. Three times the award money for Hard work added to that given for Honesty amounts to Rs. 11000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, honestly, regularity and hard work, suggest one more value which the school must include for awards.

13 If $y = sin^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{2}\right)$ find $\frac{dy}{dx}$.

OR

$If\ x = cos\,\theta + \log tan\frac{\theta}{2}\ and\ y = \sin\theta$ find $\frac{d^2y}{dx^2}\ at\ \theta = \frac{\pi}{4}$

14 Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$

15 Form a differential equation of the family of all circles having centres on x-axis and radius 1 units.

OR

Solve the differential equation: $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$.

16 Evaluate: $\int \frac{sinx - x\,cosx}{x(x + sinx)}\,dx$

OR

Evaluate: $\int \frac{1}{1+5sin^2x}\,dx$

17 Using properties of determinants, prove that: $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

18 Solve: $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

19 If $f(x) = x^2 - 5x + 7\ and\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, find $f(A)$.

SECTION –C

20 A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A. If 2 or 3 turns up, a ball is picked up from bag B. If 4, 5 or 6 turns up a ball is picked up from bag C. Bag A contains 3 red and 2 white balls; Bag B contains 3 red and 4 white balls; Bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked and a ball is drawn. If the ball drawn is red, what is the probability that it is from bag B.

21 A company has two factories located at P and Q and has three depots situated at A, B and C. The weekly requirement of the depots at A, B and C is respectively 5, 5 and 4 units , while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below

Cost (in Rs.)			
To / From	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum?

22 Let f : N → Y be a function defined by f(x) = $4x^2 + 12x + 15$, where Y = range of f. Show that f is invertible and find the inverse.

OR

Let $A = N \times N$ and $*$ be a binary operation on A defined by (a, b) $* (c, d) =$ (a + c , b + d). Show that $*$ is commutative and associative, find the identity element of $*\ on\ A$, if any.

23 Tangent to the circle $x^2 + y^2 = 4$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle. Find the minimum value of (OA + OB).

OR

A closed right circular cylinder has a volume of 2156 cubic units. What should be the radius of its base so that the total surface area may be maximum?

24 Solve the differential equation: $(3xy + y^2)dx + (x^2 + xy)dy = 0$.

25 Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$, measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.

26 Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the straight line $\frac{x}{3} + \frac{y}{2} = 1$, using integration.